International Journal of Theoretical & Applied Sciences, **1**(2): 97-101(2009)

Creep transistion in transversely isotropic circular cylinder subjected to torsion

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ABSTRACT : The transition theory of creep developed by Seth [1] has been used to derive the creep stresses in transversely isotropic circular cylinder subjected to torsion. It has been shown that the asymptotic solution through the stress- difference gives the creep stresses. Results obtained have been discussed numerically and depicted graphically. It has been observed that the value of maximum shear stress ratio for a cylinder made of transversely isotropic material is greater than that of cylinder made of isotropic material and it increases with the increase in the value of measure *n* (= 2/*N*). It has been shown that for vanishing anisotropy, the creep stresses are the same as those given by Marin [2].

Keywords : Creep Transition, Transversely Isotropic Circular Cylinder, Angle of Twist, Shear Stress Ratio

INTRODUCTION

Many authors use the incompressibility of the material in the theory of creep as the starting point for calculating stresses. The condition of incompressibility in the problems of creep deformations is one of the most important assumptions which simplify the problem. Moreover, in some cases, it is impossible to find the closed form solutions without these assumptions. It is well known that there are many materials which show compressibility effect in creep deformation. Seth [1] has developed the transition theory of creep, which does not require assumptions of incompressibility condition, and yield conditions. It utilizes the concept of generalized strain measure and asymptotic transition through the critical points of the differential system defining the deformation field. A number of problems have been solved using this measure [3-6].

In this paper, creep stresses in circular cylinder of a transversely isotropic material,subjected to torsion have been calculated using the concept of generalized strain measure.

Seth [7] has defined the generalized strain measure for uni-axial case as,

$$e = \left[\frac{1}{n}\left\{1-\left(\frac{l_0}{l}\right)^n\right\}\right]^m \qquad \text{...(1)}$$

Where n is the measure; m is the irreversibility index; and l_0, l are the initial and strained lengths respectively. For n = 0, 1, 2,-1,-2, it gives the HENCKY, SWAINGER, ALMANSI, CAUCHY and GREEN measure respectively.

GOVERNING EQUATIONS

Consider a circular cylinder of radius 'a' subjected to finite twist. The components of displacement in cylindrical co-ordinates are given by,

$$u = r(1-\beta);\ v = \propto r\,z \text{ and } w = d.z \qquad \text{...(2)}$$

Where $\beta = \beta(r)$ function of $r = \sqrt{x^2+y^2}$ only, $\propto$ is angle of twist per unit length and d is a constant.

The generalized components of strains from = n (1) are,

$$e_{rr} = \frac{1}{n^m}\left[1-(\beta+r\beta')^n\right]^m,$$

$$e_{\theta\theta} = \frac{1}{n^m}\left[1-\beta^n\right]^m, \qquad \text{...(3)}$$

$$e_{zz} = \left[\frac{D^n}{n}\right]^m\left[1-\left(\frac{\alpha r\beta}{D}\right)^n\right]^m,$$

$$e_{\theta z} = \frac{1}{n^m}\left[(\alpha\,r)^{\frac{n}{2}}\beta^n\right]^m$$

$$e_{r\theta} = e_{zr} = 0, \text{ Where } D^n = [1-(1-d)^n].$$

The stress-strain relations for transversely isotropic material are [10, 11],

$$\begin{aligned} T_{rr} &= C_{11}\,e_{rr} + (C_{11}-2C_{66})e_{\theta\theta} + C_{13}\,e_{zz}, \\ T_{\theta\theta} &= (C_{11}-2C_{66})e_{rr} + C_{11}\,e_{\theta\theta} + C_{13}\,e_{zz}, \qquad \text{...(4)} \\ T_{zz} &= C_{12}\,(e_{rr}+e_{\theta\theta}) + C_{32}\,e_{zz}, \\ T_{\theta z} &= C_{44}\,e_{\theta z}, \\ T_{zr} &= T_{r\theta} = 0. \end{aligned}$$

Using equation (3) in (4), we get transitional stresses generalized forms are,

$$T_{rr} = \frac{c_{11}}{n^m}\left[1-\{r\beta'+\beta\}^n\right]^m$$

$$+ \left[\frac{c_{11}-2c_{66}}{n^m}\right][1-\beta^n]^m + \frac{c_{12}}{n^m}D^m\left[1-\left(\frac{\alpha r\beta}{D}\right)^n\right]^m$$

$$T_{\theta\theta} = \left[\frac{c_{11}-2c_{66}}{n^m}\right]\left[1-\{r\beta'+\beta\}^n\right]^m$$

$$+ \frac{c_{11}}{n^m}[1-\beta^n]^m + \frac{c_{12}}{n^m}D^m\left[1-\left(\frac{\alpha r\beta}{D}\right)^n\right]^m$$

$$T_{zz} = \frac{c_{13}}{n^m}\left[1-\{r\beta'+\beta\}^n\right]^m$$

$$+ \frac{c_{13}}{n^m}[1-\beta^n]^m + \frac{c_{33}}{n^m}D^m\left[1-\left(\frac{\alpha r\beta}{D}\right)^n\right]^m$$

$$T_{\theta z} = \frac{c_{44}}{n^m}\left[(\alpha r)^{\frac{n}{2}}\beta^n\right]^m$$

$$T_{zr} = T_{r\theta} = 0. \qquad ...(5)$$

The equations of equilibrium are all satisfied except

$$\frac{\partial T_{rr}}{\partial r} + \frac{1}{r}[T_{rr} - T_{\theta\theta}] = 0. \qquad ...(6)$$

Using equation (5) in (6), we get a non-linear differential equation in β as,

$$P\,(1+P)^{n-1}\,\beta\frac{dP}{d\beta} \qquad ...(7)$$

Secondary creep holds when $m = 1$, then $= n$ (7) reduces to,

$$\frac{d\beta}{dP} = \frac{\left[P(1+P)^{n-1}\beta\right]}{\left[\frac{c_{11}-2c_{66}}{n}P + \frac{c_{12}}{c_{11}}(1+P)(\alpha r)^n - \frac{2c_{66}}{nc_{11}} + (1+P)^n\left(P+\frac{2c_{66}}{nc_{11}}\right)\right]} \qquad ...(8)$$

The transitional points of β in equation (8) are

$$P \to -1 \text{ and } P \to \pm\infty. \qquad ...(9)$$

The boundary conditions are

$$T_{rr} = 0 \quad \text{at} \quad r = a. \qquad ...(10)$$

$$\text{and} \int_0^a r\; T_{zz}\,dr = 0. \qquad ...(11)$$

The twisting couple W is given by

$$W = \pi\int_0^a r^2\,T_{\theta z}\,dr \qquad ...(12)$$

SOLUTION THROUGH THE PRINCIPAL STRESS

It has been shown [8, 12, 13] that the asymptotic solution through the principal stress-difference at the transition point $P \to -1$ leads to creep state.

We define the transition function R as,

$$R = T_{rr} - T_{\theta\theta}$$

$$= \frac{2C_{66}}{n}\left[\left[1-\{r\beta'+\beta\}^n\right]^m - \left[1-\beta^n\right]^m\right] \qquad ...(13)$$

Substituting the value of $\frac{dP}{d\beta}$ from $= n$(7) in logarithmic differentiation of $= n$(13) and taking the asymptotic value, we get,

$$\frac{d(\log R)}{d\beta} = \left[\frac{2C_{66}}{\beta C_{11}}\right] + \left[\frac{2mn\beta^{n-1}\{1-\beta^n\}^{m-1}}{[1-\{1-\beta^n\}^m]}\right]\cdot\left[\frac{C_{11}-C_{66}}{C_{11}}\right] \qquad ...(14)$$

Integrating $= n$ (14), we get,

$$R = Ar^{-c_2}[1-\{1-\beta^n\}^m]^{(2-c_2)} \qquad ...(15)$$

Where $c_2 = 2C$ and A is a constant of integration.

For secondary state of creep ($m = 1$), equation (15) reduces to,

$$R = T_{rr} - T_{\theta\theta} = A_1 r^{-c_2}[r]^{n(2-c_2)} \qquad ...(16)$$

Using equation (5) in equation (16), we have,

$$T_{rr} = A_1 = \frac{r^{-2n+c_2(n-1)}}{2n-c_2(n-1)} = B \qquad ...(17)$$

$$T_{\theta\theta} = \left[r^{-2n+c_2(n-1)}A_1\right]\left[\frac{1}{2n-c_2(n-1)} - 1\right] + B \qquad ...(18)$$

$$T_{zz} = \left[\frac{c_{32}}{2c_{13}}\right]$$

$$\cdot\left[r^{-2n+c_2(n-1)}A_1\right]\left[\frac{1}{2n-c_2(n-1)}-1\right] + B\frac{c_{33}}{c_{12}}$$

$$+\left[\frac{c_{12}^2-(c_{11}-c_{66})c_{33}}{nc_{13}}\right](2-\beta^n) \qquad \text{...(19)}$$

Using boundary conditions (10) and (11) in equation (17), we get the value of A_1 and B,

$$B = -A\times\left[\frac{a^{-2n+c_2(n-1)}}{2n-c_2(n-1)}\right]$$

Putting the value of B in (17), (18) and (19), we get transitional creep stresses as,

$$T_{rr} = A_1\times\left[\frac{a^{-2n+c_2(n-1)}}{2n-c_2(n-1)}\right]\left[\left(\frac{r}{a}\right)^{-2n+c_2(n-1)}-1\right] \qquad \text{...(20)}$$

$$T_{\theta\theta} = A_1\times\left[\frac{a^{-2n+c_2(n-1)}}{2n-c_2(n-1)}\right]$$

$$\left[\{1-2n+c_2(n-1)\}\times\left(\frac{r}{a}\right)^{-2n+c_2(n-1)}-1\right] \qquad \text{...(21)}$$

$$T_{zz} = A_1\times\left[\frac{c_{32}}{2c_{13}}\right]\left[\frac{a^{-2n+c_2(n-1)}}{2n-c_2(n-1)}\right]$$

$$\left[\{2-2n+c_2(n-1)\}\times\left(\frac{r}{a}\right)^{-2n+c_2(n-1)}-2\right]$$

$$+\left[\frac{c_{12}^2-(c_{11}-c_{66})c_{32}}{nc_{12}}\right](2-\beta^n) \qquad \text{...(22)}$$

The value of A_1 is obtained from (11) and (22), we have,

$$A_1 = \frac{2C_{66}}{n}\,[2-2n+c_2(n-1)]\,a^{[2n+c_2(n-1)]} \qquad \text{...(23)}$$

The asymptotic value of β is found by solving equations (13) and (15), is,

$$\beta^n = \left[\frac{2C_{66}}{nA}\right]^{\left(\frac{1}{1-c_2}\right)}\times[r]^{\left(\frac{c_2}{1-c_2}\right)} \qquad \text{...(24)}$$

Using equation (24) in (4), we get shearing stress as,

$$T_{\theta z} = \frac{c_{44}}{n}(\alpha)^{\frac{n}{2}}\left[\frac{2C_{66}}{nA}\right]^{\left(\frac{1}{1-c_2}\right)}\times[r]^{\left(\frac{n}{2}+\frac{c_2}{1-c_2}\right)} \qquad \text{...(25)}$$

The twist couple is given from equation (12) as,

$$W = 2\pi\int_0^a r^2\,T_{\theta z}\,dr$$

$$= \frac{\left[\frac{2\pi}{n}c_{44}\left[\frac{2C_{66}}{nA}\right]^{\left(\frac{1}{1-c_2}\right)}[a]^{\left(\frac{n}{2}+\frac{c_2}{1-c_2}+3\right)}\right]}{\left[\frac{n}{2}+\frac{c_2}{1-c_2}+3\right]} \qquad \text{...(26)}$$

The relation betweeen W and $T_{\theta z}$ is

$$T_{\theta z} = \frac{1}{4}\left[\frac{n}{2}+\frac{c_2}{1-c_2}+3\right]\left[\frac{2W}{\pi a^2}\right]\left[\frac{r}{a}\right]^{\left(\frac{n}{2}+\frac{c_2}{1-c_2}\right)} \qquad \text{...(27)}$$

The maximum shearing stress occurs at $r = a$, is given by

$$T_{\max} = \frac{1}{4}\left[\frac{n}{2}+\frac{c_2}{1-c_2}+3\right]T_e \qquad \text{...(28)}$$

Where $T_e = \dfrac{2W}{\pi a^2}$ = elastic maximum shear stress.

ISOTROPIC MATERIAL

For isotropic materials, the material constants reduces to two only [11], *i.e.*,

$$C_{11} = C_{22} = C_{33},$$

$$C_{12} = C_{21} = C_{13} = C_{31} = C_{23} = C_{32} = (C_{11} = -2C_{66}).$$

In terms of Lame's constant and , these can be written as,

$$C_{12} = \lambda,\ C_{66} = \frac{1}{2}(C_{11}-C_{12}) \equiv \mu \text{ and } C_{11} = \lambda + 2\mu. \qquad \text{...(29)}$$

Where $c = \dfrac{2\mu}{\text{EMBED Equation.3} \quad +\mu}$.

Creep transitional stresses (20), (21) and (22) for isotropic material becomes,

$$T_{rr} = A_1 \times \left[\frac{a^{-2n+c(n-1)}}{2n-c(n-1)}\right]\left[\left(\frac{r}{a}\right)^{-2n+c(n-1)} - 1\right] \quad ...(30)$$

$$T_{\theta\theta} = A_1 \times \left[\frac{a^{-2n+c(n-1)}}{2n-c_2(n-1)}\right]$$

$$\left[\{1-2n+c(n-1)\}\times\left(\frac{r}{a}\right)^{-2n+c(n-1)} - 1\right] \quad ...(21)$$

$$T_{zz} = A_1 \times \left[\frac{1}{2-c}\right]\left[\frac{a^{-2n+c(n-1)}}{2n-c(n-1)}\right]$$

$$\left[\{2-2n+c(n-1)\}\times\left(\frac{r}{a}\right)^{-2n+c(n-1)} - 2\right]$$

$$+\left[\frac{(3-2c)}{(1-c)}\right]\left[\frac{\mu}{n}\right](2-\beta^n) \quad ...(32)$$

Where $A_1 = \left(2\frac{\mu}{n}\right)[2-2n+c(n-1)](a)^{-2n+c(n-1)}$

The shearing stress for isotropic material becomes,

$$T_{\theta z} = \left[\frac{n}{2}+\frac{c}{1-c}+3\right]\left[\frac{2W}{\pi a^2}\right]\left[\frac{r}{a}\right]^{\left(\frac{n}{2}+\frac{c}{1-c}\right)} \quad ...(33)$$

and maximum shearing stress for isotropic material is given by,

$$T_{\max} = \frac{1}{4}\left[\frac{n}{2}+\frac{c}{1-c}+3\right]T_e \quad ...(34)$$

For incompressible material *i.e.*, $c \to 0$, then

$$T_{\max} = \frac{1}{4}\left[\frac{n}{2}+3\right]T_e \quad ...(35)$$

This expression of shearing stress is given by Marin[2] provided we put $n = \frac{2}{N}$.

NUMERICAL ILLUSTRATION AND DISCUSSION

As a numerical example, elastic constants C_{ij} have been given in Table 1 for transversely isotropic material [10] (C_2 = 0.64, Magnesium) and isotropic material [14] (Brass, C = 0.50, σ = 0.33).

Table : 1. ELASTIC CONSTANTS (In units of 10^{10} N/m^2).

	C_{44}	C_{11}	C_{12}	C_{13}
Transversely Isotropic Material (C_2 = 0.64, **Magnesium**)	1.64	5.97	2.62	2.17
Isotropic Material (C = 0.50, **Brass**)	0.99997	3.0	1.0	1.0

For calculating creep stress distribution based on above analysis the following values of measure n have been taken

$$n = \frac{1}{3}, 2, 3 \ (i.e.\ N = 2/n = 6, 1, 0.66)$$

Curves have been drawn in Figures 1, 2, 3; between shear stress ratio and radii ratio (r/a) for transversely isotropic material (C_2 = 0.64, Magnesium) and isotropic material (Brass, C = 0.50, σ = 0.33) and c = 0 represents the curve for isotropic incompressible material.

It can be seen from Figures 1, 2, 3; that cylinder made of transversely isotropic material, the shear stress is maximum at outer surface as compare to cylinder made of isotropic material which further increases with the increase in measure n.

For n = 2 & c = 0, elastic shear stress curve is obtained. The value n = 1/3 is considered to take account of the classical results obtained by Marin [2].

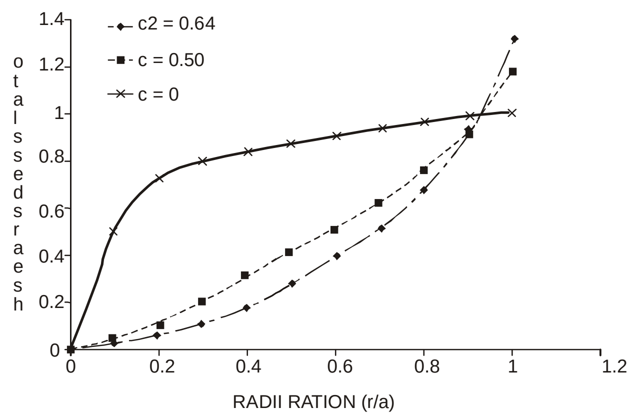

Fig.1.

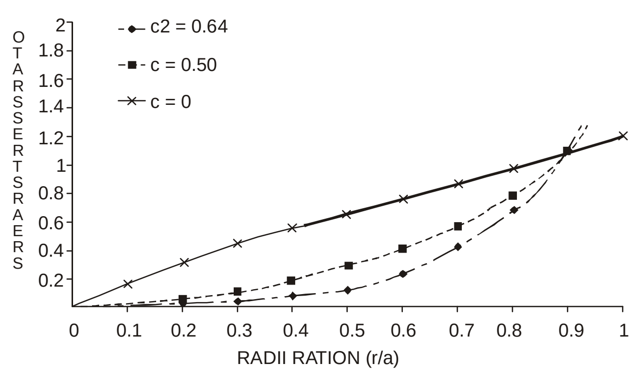

Fig.2.

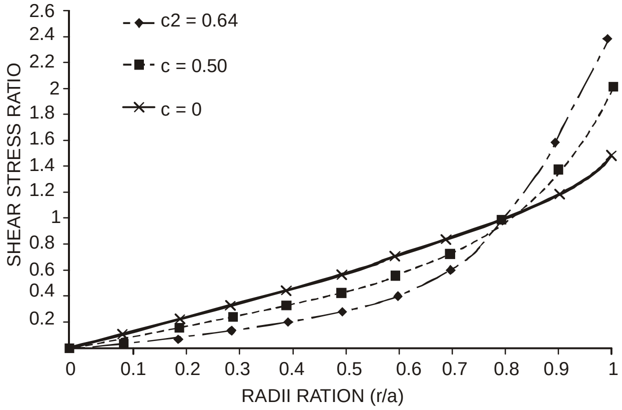

Fig.3.

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