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# Some results on fixed point theorem in Dislocated Quasi Metric Spaces

***Manvi Kohli, Rajesh Shrivastava and Manoj Sharma****

*Govt. Science and Commerce Benazir College, Bhopal (MP) INDIA*
**Sagar Inst. of Research and Tech., Bhopal (MP) INDIA*

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**ABSTRACT : In this paper we have proved fixed point theorem for continuous contraction mappings in dislocated Quasi Metric Spaces. Aslo we obtain a common fixed point theorem for a pair of mappings in Dislocated Metric Spaces.**

**Keywords :** Dislocated quasi matrices fixed point.

## INTRODUCTION

Banach [1992] proved Fixed Point Theorem for Contraction Mappings in Complete Matrix Space. It is well known as a Banach Fixed Point Theorem. Dass and Gupta [1] generalized Banach's Contraction Principle in Metric Space. Also Rhoads [1977] introduced a partial ordering for various Definitions Contractive Mappings. This objective of the note is to prove some fixed point theorem for continuous contraction mapping defined by Dass and Gupta [1] and Rhoades [4] in Dislocated Quasi Metric Spaces.

## PRELIMINARIES

**Definition 1 [3] :** Let $X$ be a nonempty set and let $d : X\text{x}X \to [0, \infty]$ be a function satisfying following conditions.

(*i*) $d(x, y) = d(y, x) = 0$ implies $y = x$

(*ii*) $d(x, y) < d(x, z) + d(z, y)\ \forall\ x, y, z \in X$

Then $d$ is called Dislocated Quasi Metric Space on $X$. If $d$ satisfies $d(x, y) = d(y, x)$ then it is called dislocated metric space.

**Definition 2 [3] :** A Sequence $[X_n]$ is $dq$ Metric Space (Dislocated Quasi Metric Spaces) $(X, d)$ is called Cauchy Sequence if for given $\varepsilon > 0$, $\exists\ a\ n_0 \in N$ such that $\forall\ m, n > n_0$

$\Rightarrow\ d(x_m, x_n) < \varepsilon$ or $d(x_n, x_m) < \varepsilon$

*i.e.*, $\min\ \{d(x_m, x_n), d(x_n, x_m)\} < \varepsilon$

**Definition 3 [3] :** A Sequence $[X_n]$ dislocated Quasi Convergence to $x$ if

$1t\ n \to \infty\ d(x_n, x) = 1t\ n \to \infty\ d(x, x_n) = 0$

In this case $x$ is called a $dq$ limit of $[x_n]$ we write $x_n \to x$.

**Definition 4 [3] :** A $dq$ Metric Space $(X, d)$ is called complete if every Cauchy Sequence in it is a $dq$ convergent.

**Definition 5 [3] :** Let $(X, d)$ and $(Y, d)$ be $dq$ Metric Spaces and Let $f : X \to Y$ be a function. Then $f$ is continuous to $x_0 \in X$, if for each sequence $[x_n]$ which is $d_1 - q$ convergent to $x_0$ in $X$, the sequence $[f(x_n)]$ is $d_2 - q$ convergent to $f(x_o)$ in $Y$.

**Definition 6 [3] :** Let $(X, d)$ be a $dq$ Metric Space. A map $T : X \to X$ is called contraction if there exists $0 < x < 1$ such that

$$d(Tx, Ty) < \lambda\ d(x, y) \forall x, y \in X$$

**Theorem 1 :** Let $(X, d)$ be a $dq$ Metric and let $T : X \to X$ be continuous contracting mapping. Then $T$ has a unique fixed point.

## MAIN RESULT

**Theorem 1 :** Let $(X, d)$ be a $dq$ Metric Space and let $T : X \to X$ be continuous mapping satisfying the following condition.

$$d(Tx, Ty) < \alpha \frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)} + \beta d(x, y) + \gamma d(y, Ty)$$

$$\forall\ x, y \in X, \alpha > 0, \beta > 0, r > 0\ \alpha + \beta + \gamma < 1$$

Then $T$ has a unique fixed point.

**Proof :** Let $[X_n]$ be a sequence in $X$ defined as follows. Let $x_0 \in X$, $T(x_0) = x_1$, $T(x_1) = x_2$, $T(x_3) = x_4 \ldots T(x_n) = x_{n+1}$.

Consider,

$$d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n)$$

$$< \alpha \frac{d(x_n,Tx_n)[1+d(x_{n-1},Tx_{n-1})]}{[1+d(x_{n-1,}x_n)]}$$

$$+ \beta d(x_{n-1}, x_n) + \gamma d(x_n, Tx_n) \qquad \ldots(i)$$

$$d(x_n, x_{n+1}) < \alpha \frac{d(x_n,x_{n+1})[1+d(x_{n-1},x_n)]}{[1+d(x_{n-1,}x_n)]}$$

$$+ \beta d(x_{n-1}, x_n) + \gamma d(x_n, x_{n+1})$$

Therefore

$$d(x_n, x_{n+1}) - \alpha d(x_n, x_{n+1}) - \gamma d(x_n, x_{n+1}) < \beta d\ (x_{n-1}, x_n)$$

$$\Rightarrow (1 - \alpha - \gamma)\ d(x_n, x_{n+1}) < \beta\ d(x_{n-1}, x_n)$$

$$\Rightarrow d(x_n, x_{n+1}) < \frac{\beta}{1-\alpha-\gamma}\ d(x_{n-1,} x_n)$$

Let $\quad \delta = \dfrac{\beta}{1-\alpha-\gamma}$ with $0 < \delta < 1$

Then $d(x_n, x_{n+1}) < \delta\, d(x_{n-1,} x_n)$

On further decomposing we get

$$d(x_{n-1}, x_n) < \delta\, d(x_{n-2,} x_{n-1})$$

and finally we can write

$$d(x_n, x_{n+1}) < \delta^2\, d(x_{n-2,} x_{n-1}).$$

On continuing this process n times

$$d(x_n, x_{n+1}) < \delta^2\, d(x_{0,} x_1)$$

Since $0 < \delta < 1$ and $n \to \infty$, $\delta^n \to 0$.

Hece $[X_n]$ is a *dq* sequence in the complete dislocated Quasi Metric Space $X$.

Thus $[X_n]$ dislocated Quasi sequence converges to come $t_0$. Since $T$ is continuous we have

$$T(t_0) = 1t_{n \to \infty}\, T(X_n) = 1t_{n \to \infty}\, x_{n+1} = t_0$$

Thus $T(t_0) = t_0$

Thus $T$ has a fixed point.

## Uniqueness

Let $x$ be a fixed point of $T$. Then by given condition we have

$$d(x, x) = d(Tx, x) < (\alpha + \beta + \gamma)\, d(x, x)$$

Which gives $d(x, x) = 0$, Since $0 < (\alpha + \beta + \gamma) < 1$ and $d(x, x) > 0$.

Thus $d(x, Tx) =$ if $x$ is a fixed point of $T$.

Let $x, y \in X$ be fixed points of $T$, *i.e.*, is $Tx = x$; $Ty = y$.

Then by condition 3.1 $d(x, y) = d(Tx, Ty) < \beta\, d(x, y)$ which gives $d(x, y) > 0$, Since $0 < \beta < 1$ and $d(x, y) = 0$.

Similarly $d(y, x) = 0$ and hence $x = y$. Thus fixed point of $T$ is unique.

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