International Journal of Theoretical & Applied Sciences, 2(1): 29-33(2010) ISSN : 0975-1718

Free convection MHD flow past a moving vertical porous surface with gravity modulation at constant heat flux

Satish C. Rajvanshi and Baljinder S. Saini

Deptt. of Applied Sci., S.V. Institute of Emerging Technology, Gurukul Vidyapeeth Campus, Banur, (PB) INDIA

(*Received 6 Jan., 2010, Accepted 25 Jan., 2010*)

ABSTRACT : The effects of fluctuating gravitational field on free convection MHD flow past a uniformly moving infinite vertical porous plate with time-dependent suction velocity in a porous medium have been analyzed. A constant heat flux is prescribed on the plate. The gravitational field is assumed in the form $g = g_0 + g_1 \cos(\omega^* t^*)$.The governing equations are solved by perturbation technique. Fluid velocity and fluid temperature shows remarkable change with change in gravitational field. Small increase in gravity modulation parameter shows significant increase in amplitude of skin-friction and has insignificant decreasing effect on phase of skin-friction.

Keywords : MHD, Gravity modulation, Heat Flux, Free convection, Oscillatory suction, Porous medium

NOMENCLATURE

B_0 = magnetic field intensity
C = dimensionless species concentration in the fluid
C^* = species concentration in the fluid
C_p = specific heat at constant pressure
c_∞^* = species concentration in free stream
c_w^* = species concentration at the wall
D = chemical molecular diffusivity
Ec = Eckert number
g = acceleration due to gravity
g_0 = constant gravity level
g_1 = the amplitude of oscillating component of acceleration due to gravity
Gc = modified Grash of number
Gr = Grash of number
K = dimensionless permeability of porous medium
K_0 = permeability constant of porous medium
K^* = permeability of porous medium
M = magnetic parameter
p = pressure
$\Pr$ = Prandtl number
q = Heat flux per unit area
Sc = Schmidt number
T^* = temperature
T_∞^* = temperature of fluid in free stream
t = dimensionless time
t^* = time
u = dimensionless velocity of fluid
u^* = velocity of fluid
U = dimensionless velocity of the moving vertical porous plate
U^* = velocity of the moving vertical porous plate
V = suction velocity
V_0 = suction velocity constant of the fluid through the porous surface
β = coefficient of thermal expansion
β^* = coefficient of thermal expansion with concentration
λ = thermal conductivity
ε = a constant ($0 < \varepsilon << 1$)
ϕ = phase difference for skin friction
κ = thermal conductivity
μ = viscosity
υ = kinematic viscosity
ρ = density of the fluid
σ = electric permeability
θ = dimensionless temperature
ω = dimensionless frequency of gravitational oscillation
ω^* = frequency of gravitational oscillation
τ = skin friction
τ_m = mean skin friction

INTRODUCTION

The study of porous media is widely used in high temperature heat exchangers, turbine blades and jet nozzles. Porous media are useful in diminishing the natural free convection which would be intense on a vertical heated surface. To make heat insulation of surface more effective, it is necessary to study the free convection flow through a porous medium. Unsteady oscillatory free convection flows play an important role in chemical engineering, turbo machinery and aerospace technology. The free convection effect on the flow of an ordinary viscous fluid past an infinite, vertical porous plate with constant suction and constant heat flux was investigated [1]. Magnetic field effects on the free convection and mass transfer flow through a porous medium with periodic suction and constant heat flux [2]. Saini and Sharma [3] have studied heat and mass transfer in MHD flow past a vertical plate embedded in porous medium. Soundlgekar and Patil [4] have studied

Stokes problem for infinite vertical plate with constant heat flux. They observed that with increasing time and Grashof number the velocity of the fluid increases.

The effect of gravity modulation and magneto-hydrodynamics on a free convection flow is useful in space technology. It needs special attention on forces involving vibrations that occur due to interaction of several phenomena. In space vehicle, there are transient perturbations to the gravity field at a point. An excellent account of this physical feature has been described by Li [5]. Sharidan and Pop [6] have studied g-jitter fully developed combined heat and mass transfer by mixed convection flow in a vertical channel.

In the space lab and Shuttle environments, the existence of perturbative accelerations, characterized by a broad frequency spectrum, is well known. These perturbations are caused by mechanical vibrations, orbiter maneuvers and crew activities. These cannot be totally eliminated from the space environment. A computational study for the investigation of gravity modulation effects in thermally driven cavity flows at terrestrial and microgravity environments have been analyzed by Biringen [7]. The two-dimensional, time-dependent Navier-Stokes equations are numerically integrated by a time-split method using direct matrix solvers.

Numerical analysis of thermosolutal flows in a cavity with gravity modulation effects have been studied by Jue and Ramaswamy [8]. They used semi-implicit projection finite element method to solve the transient Navier–Stokes, energy and species concentration equations. A finite element study of double-diffusive convection driven by g-jitter in a microgravity environment is studied by Shu *et. al.*, [9]. They have included the use of idealized single-frequency and multifrequency g-jitter as well as the real g-jitter data taken during an actual Space Shuttle flight. Numerical study indicates that with an increase in g-jitter force, the nonlinear convective effects become much more obvious, which in turn drastically change the concentration fields. The simulated results using the g-jitter data taken during space flights show that both the velocity and concentration become random, and follow approximately the same pattern as the g-jitter perturbations. The effect of time-periodic temperature/ gravity modulation at the onset of magneto-convection in weak electrically conducting fluids with internal angular momentum is investigated by Siddheshwar and Pranesh [10].

The aim of the paper is to investigate the effect of gravity modulation with different amplitudes and frequencies on free convection of a viscous fluid past a uniformly moving infinite vertical porous plate with constant heat flux and varying suction velocity. Governing equations have been solved with regular perturbation method. The variation in gravity modulation and magnetic parameter make significant change in skin-friction.

FORMULATION OF THE PROBLEM

We consider MHD incompressible viscous fluid past a uniformly moving infinite vertical porous plate. Plate is bounded by a porous medium of time dependent permeability and suction velocity. The x^*-axis is taken along the plate and y^*-axis is taken normal to the plate. A uniform magnetic field normal to the direction of flow is introduced. The magnetic Reynolds number is taken to be very small so that the induced magnetic field can be neglected in comparison to the applied magnetic field. The temperature difference between the wall and the medium develops buoyancy force which induces the basic flow. Initially the plate as well as fluid are assumed to be at the same temperature and the concentration of species is very low.

Since the plate is considered infinite in the x^*-direction, all physical quantities are independent of x^* and are function of y^* and t^* only. Under these assumptions flow is governed by the following set of equations

$$\frac{\partial V^*}{\partial y^*} = 0 \qquad \text{...(1)}$$

$$\frac{\partial u^*}{\partial t^*} + V^* \frac{\partial u^*}{\partial y^*} = g\,\beta\,(T^* - T^*_\infty) + g\beta^*\,(C^* - C^*_\infty) + \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} - \upsilon \frac{u^*}{K^*} - \frac{\sigma}{\rho^*} B_0^2 u^* \qquad \text{...(2)}$$

$$\frac{\partial T^*}{\partial t^*} + V^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho^* C_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}}\right) + \frac{\upsilon}{C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 \qquad \text{...(3)}$$

$$\frac{\partial C^*}{\partial t^*} + V^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} \qquad \text{...(4)}$$

The suction velocity on the vertical plate is imposed in the form

$$V^* = -V_0^*\,(1 + \varepsilon\, e^{i\omega^* t^*}) \qquad \text{...(5)}$$

The relevant boundary conditions are at $y^* = 0$:

$$u^* = U^*,\ V = -V_0^*\,(1 + \varepsilon\, e^{i\omega^* t^*}),$$

$$\frac{\partial T^*}{\partial y^*} = -\frac{q}{\lambda},$$

$$C^* = C_w^* + \varepsilon(C_w^* - C_\infty^*)e^{i\omega^* t^*} \qquad \text{...(6a)}$$

as $y^* \to \infty$:

$$u^* \to 0,\ T^* \to T^*_\infty,\ C^* \to C^*_\infty \qquad \text{...(6b)}$$

The (*) stands for dimensional quantities. The subscript (∞) denotes the free stream condition.

The permeability of the porous medium is taken in the form

$$K^* = K_0\,(1 + \varepsilon\, e^{i\omega^* t^*}) \qquad ...(7)$$

The time-dependent gravitational acceleration is assumed in the form $g = g_0 + g_1 \cos \omega^* t^*$, where g_0 is the constant gravity level in the environment, g_1 is the amplitude of the oscillating component of acceleration and ω^* is the frequency of gravitational oscillation. The gravitational acceleration is rewritten in the form

$$g = g_0 + g_1 e^{i\omega^* t^*} \qquad ...(8)$$

The following non-dimensional quantities are introduced

$$y = \frac{y^* V_0^*}{\upsilon},\; t = \frac{t^* V_0^{*2}}{4\upsilon},\; \omega = \frac{4\upsilon\omega^*}{V_0^{*2}},\; u = \frac{u^*}{V_0^*},$$

$$U = \frac{U^*}{V_0^*},\; C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*},\; \theta = \frac{(T^* - T_\infty^*)\, V_0^* \lambda}{q\upsilon},$$

Grash of Number $\quad Gr = \dfrac{\upsilon^2 \beta\, g_0 q}{V_0^{*4} \lambda}$,

Prandtl Number $\quad Pr = \dfrac{\upsilon}{(\kappa/\rho\, C_p)}$,

Magnetic parameter $\quad M = \dfrac{B_0}{V_0^*}\sqrt{\dfrac{\sigma\upsilon}{\rho^*}}$,

Eckert number $\quad Ec = \dfrac{V_0^{*3}\lambda}{C_p\, q\upsilon}$,

Modified Grashof Number $\quad Gc = g_0 \beta^*_V \dfrac{(C_w^* - C_\infty^*)}{V_0^{*3}}$,

Schmidt number $\quad Sc = \dfrac{\upsilon}{D}$,

Permeability parameter $\quad K = \dfrac{V_0^{*2} K^*}{\upsilon^2} \qquad ...(9)$

The physical variables used herein are defined in Nomenclature.

Equations (1) through (9) take the following non-dimensional form

$$\frac{\partial V}{\partial y} = 0 \qquad ...(10)$$

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1 + \varepsilon e^{i\omega t})\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + (1 + \varepsilon\alpha\, e^{i\omega t})(\theta\, Gr + C\, Gc) - M^2 u - \frac{u}{K_0 (1 + \varepsilon\, e^{i\omega t})} \qquad ...(11)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - (1 + \varepsilon e^{i\omega t})\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 \qquad ...(12)$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - (1 + \varepsilon e^{i\omega t})\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \qquad ...(13)$$

where, $\quad g_1 = \varepsilon\alpha g_0$

The boundary conditions in the dimensionless form are

$$\left.\begin{array}{llll} u = U, & \dfrac{\partial\theta}{\partial y} = -1, & C = 1 + \varepsilon e^{i\omega t} & \text{at } y = 0 \\ u \to 0, & \theta \to 0, & C \to 0 & \text{as } y \to \infty \end{array}\right\} \qquad ...(14)$$

SOLUTION OF THE GOVERNING EQUATIONS

For small amplitude oscillation ($0 < \varepsilon << 1$), the velocity, temperature θ and concentration C are represented as

$$(u, \theta, C)\,(y, t) = (u_0, \theta_0, C_0)\,(y) + \varepsilon(u_1, \theta_1, C_1)\,(y) e^{i\omega t} \qquad ...(15)$$

Substituting (15) in (11) to (13) and separating steady and unsteady components, we have

$$u_0'' + u_0' - \left(M^2 + \frac{1}{K_0}\right) u_0 = Gr\theta_0 - GcC_0 \qquad ...(16)$$

$$u_1'' + u_1' - \left(\frac{i\omega}{4} + \frac{1}{K_0} + M^2\right) u_1 = -u_0'' - 2u_0' - (\theta_0 + \theta_1)\, Gr - (C_0 + C_1)\, Gc - \alpha(\theta_0\, Gr + C_0\, Gc) + M^2 u_0 \qquad ...(17)$$

$$\theta_0'' + \Pr\theta_0' = -Ec\Pr\,(u_0')^2 \qquad ...(18)$$

$$\theta_1'' + \Pr\theta_1' - \frac{i\omega\Pr}{4}\theta_1 = -\Pr\theta_0' - 2Ec\Pr u_0' u_1' \qquad ...(19)$$

$$C_0'' + ScC_0' = 0 \qquad ...(20)$$

$$C_1'' + Sc\,C_1' - \frac{i\omega Sc}{4} C_1 = -Sc\, C_0' \qquad ...(21)$$

where prime denotes derivative with respect to y.

The corresponding boundary conditions (14) reduce to the form

$$\left.\begin{array}{lllllll} u_0 = U, & u_1 = 0, & \theta_0' = -1, & \theta_1' = 0, & C_0 = 1, & C_1 = 1, & \text{at } y = 0 \\ u_0 \to 0, & u_1 \to 0, & \theta_0 \to 0, & \theta_1 \to 0, & C_0 \to 0, & C_1 \to 0, & \text{as } y \to \infty \end{array}\right\} \qquad ...(22)$$

The equations (16) to (19) are still coupled for the variables u_0, u_1 and θ_0, θ_1. These equations have been solved for $Ec << 1$, using perturbation technique in the following form

$$(u_0, \theta_0, u_1, \theta_1) = (u_{00}, \theta_{00}, u_{10}, \theta_{10}) + Ec\,(u_{01}, \theta_{01}, u_{11}, \theta_{11}) \quad ...(23)$$

Finally, we have

$$u(y, t) = u_{00}(y) + u_{01}(y)\,Ec + \varepsilon\,[u_{10}(y) + u_{11}(y)\,Ec]\,e^{i\omega t} \quad ...(24)$$

$$\theta(y, t) = \theta_{00}(y) + \theta_{01}(y)\,Ec + \varepsilon\,[\theta_{10}(y) + \theta_{11}(y)\,Ec]\,e^{i\omega t} \quad ...(25)$$

$$C(y, t) = C_0(y) + \varepsilon C_1(y)\,e^{i\omega t} \quad ...(26)$$

The analytical expressions have not been recorded here for the sake of brevity.

RESULTS AND DISCUSSION

We discuss velocity field, temperature field, concentration field and skin-friction by assigning numerical values to various parameters. Prandtl number for air and water at freezing point are taken as 0.71 and 11.4 respectively. The values of Schmidt number *Sc* for Hydrogen, Ammonia, Helium and Water-vapor are 0.24, 0.78, 0.30, and 0.60 respectively. We consider gravity modulation parameter $\varepsilon\alpha = 10$. The value of Eckert number *Ec* is taken 0.001 and $\varepsilon = 0.005$. The values of Grashof number *Gr*, Modified Grashof number *Gc* and permeability parameter K_0 are selected arbitrarily.

A. Velocity profiles

Fig.1 represents the velocity profiles for various values of *Gr*, *Gc*, *Sc* and *U*. The velocity component near the plate increases with increase in Grashof number *Gr*, Modified Grashof number *Gc* and *U*, and after attaining a maximum value, it start decreasing gradually. The magnitude of velocity decreases with increase in Schmidt number *Sc*. Increase in *Gr* has more impact on the increase of velocity component as compared to increase in *Gc*. Fig.2 shows the velocity profiles for various values of Magnetic parameter *M* and gravity modulation parameter $\varepsilon\alpha$. It is seen that fluid velocity decreases with increase in Magnetic parameter *M*, while it increases with increase in gravity modulation parameter $\varepsilon\alpha$. The velocity component shows more variation in the vicinity of the plate and then decreases exponentially far away from the plate.

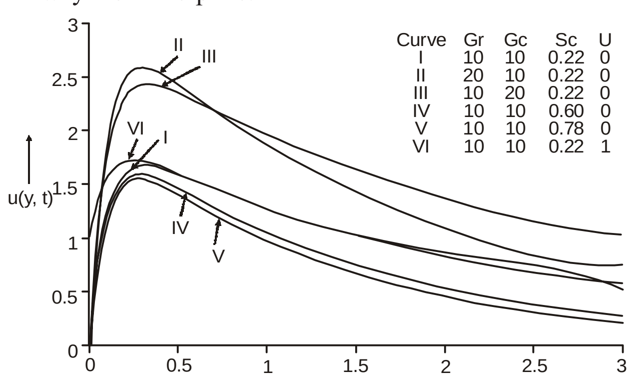

Fig.1. Velocity profiles versus y for Pr = 0.71, M = 10, εα = 10 and ωt = π/4.

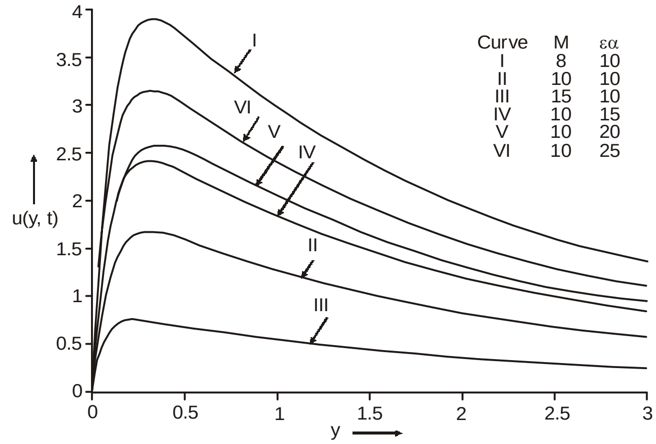

Fig.2. Velocity profiles versus y for Pr = 0.71, Sc = 0.22, Gr = 10, Gc = 10, ω = 30 and ωt = π/4.

B. Temperature profiles

Fig.3 exhibits the variation of fluid temperature for different values of Prandtl number *Pr*, Gravity modulation parameter $\varepsilon\alpha$, Magnetic parameter *M* and velocity of moving plate *U*. It is seen that with increase in Prandtl number *Pr* fluid temperature decreases more rapidly for water at freezing point (*Pr* = 11.4) in comparison to air (*Pr* = 0.71). Fluid temperature decreases with increase in *U* and Magnetic parameter *M*, while it increases with increase in gravity modulation parameter $\varepsilon\alpha$.

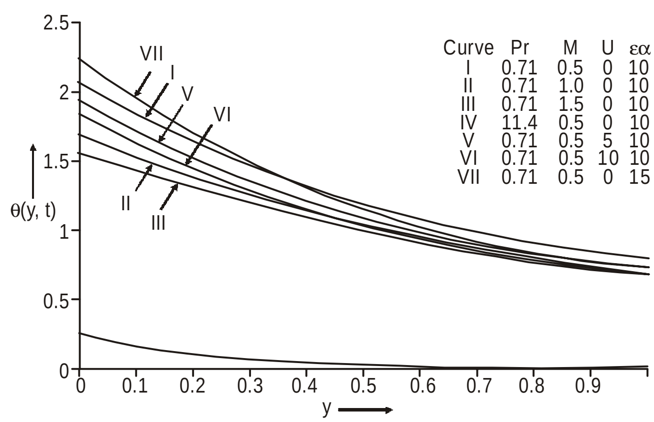

Fig.3. Temperature profiles versus y for Sc = 0.22, Gr = 10, Gc = 10, ω = 30 and ωt = π/4.

Fig.4 indicates the effects of *Gr*, *Sc*, *Gc* and K_0 on temperature profiles for $\omega = 30$, Pr = 0.71, $\varepsilon\alpha = 10$ at time $t = \pi/4\omega$. It is seen from the figure that with increase in *Gr*, *Gc*, and K_0, fluid temperature increases; while it decreases with increase in *Sc*. Temperature profiles II and III crosses at $y \approx 0.0996$. The quantitative effect of K_0 on fluid temperature is small.

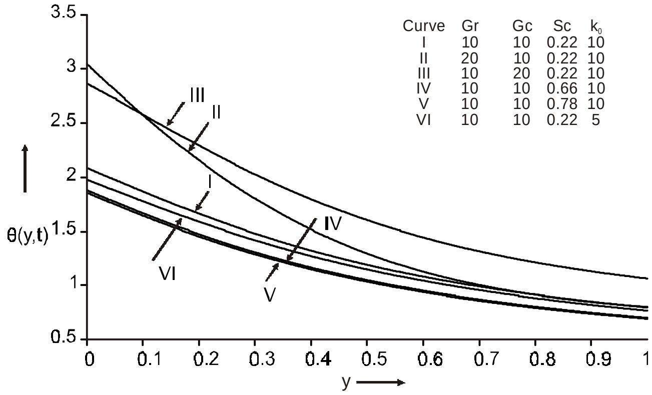

Fig.4. Temperature profiles versus y for Pr = 0.71, M = 0.5, Ec = 0.001 and ωt = π/4.

C. Concentration profiles

The effect of Schmidt number *Sc* on concentration profiles for $\omega = 30$, $\varepsilon = 0.005$, and Pr = 0.71 at time $t = \pi/4\omega$ is shown by Fig.5. It is observed that concentration profiles decreases with increase in Schmidt number *Sc*, the concentration field falls slowly and steadily. The gravity modulation and magnetic parameter *M* do not have any impact on the concentration profiles.

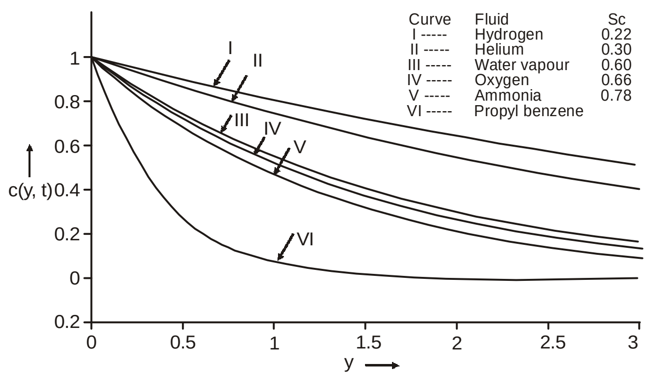

Fig.5. Concentration profiles for ω = 30, ε = 0.005 and ωt = π/4.

D. Skin-Friction

The skin-friction in the non-dimensional form on the plate $y = 0$ is given by

$$\tau = \frac{\tau^*}{\rho V_0^{*2}} = \tau_m + \varepsilon|N| \cos(\omega t + \phi) \qquad ...(27)$$

where τ_m is the mean skin friction; $\varepsilon|N|$ and ϕ are amplitude and phase difference of the fluctuating component.

Fig.6 shows the effect of magnetic parameter *M,* Schmidt number *Sc*, velocity of the moving vertical porous plate *U* and gravity modulation parameter $\varepsilon\alpha$ on skin friction τ for $Pr = 0.71$, $Gr = 5$, $Gc = 3$, $Ec = 0.001$, $\omega = 10$ and $\varepsilon = 0.005$ at $\omega t = \pi/4$ versus K_0. It is seen that τ increases with increase in gravity modulation parameter $\varepsilon\alpha$, while it decreases with increase in magnetic parameter *M*, Schmidt number *Sc* and the velocity of the moving vertical porous plate *U*. With increase in permeability parameter K_0, there is an increase in τ. There is a significant difference in the value of τ with small change in gravity modulation parameter $\varepsilon\alpha$.

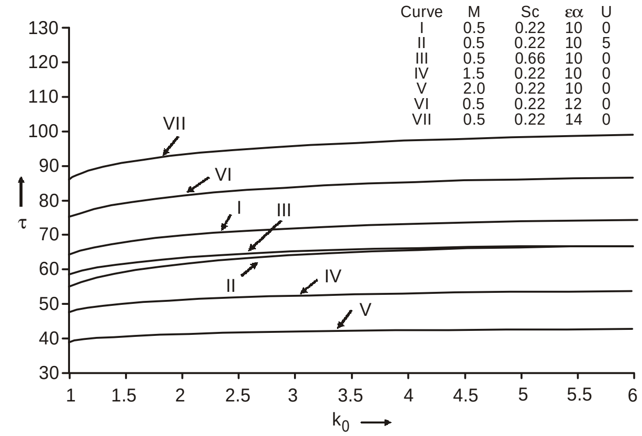

Fig.6. Skin-friction coefficient versus k_0 for Pr = 11.4, Gr = 5, Gc = 3, ω = 10 and ωt = π/4.

CONCLUSIONS

It is observed that the velocity component near the plate increases with increase in *Gr*, *Gc*, *U* and gravity modulation parameter $\varepsilon\alpha$, while it decreases with increase in *Sc* and magnetic parameter *M*. Fluid temperature decreases with increase in Prandtl number Pr, *U*, *M* and Schmidt number *Sc*, while it increases with increase in gravity modulation parameter $\varepsilon\alpha$, *Gr*, *Gc* and K_0. It is seen that skin friction coefficient increases with increase in gravity modulation parameter $\varepsilon\alpha$, while it decreases with increase in magnetic parameter *M*, with increase in permeability parameter K_0, there is an increase in skin friction coefficient. Small change in gravity modulation parameter $\varepsilon\alpha$ effects skin friction coefficient significantly.

REFERENCES

[1] P.R. Sharma, *Journal of Physics*, **25:** 162(1992).

[2] M. Acharya, G.C. Das and L.P. Singh, Indian *J. Pure Appl. Math.*, **31**(1)**:** 1(2000).

[3] B.S. Saini and P.K. Sharma, Ganita Sandesh, **20**(2)**:** 203(2006).

[4] V.M. Soundalgekar and M.R. Patil, Astrophysics and Space Science, **70**(1)**:** 179(1980).

[5] B.Q. Li, Int. *J. Heat Mass Transfer*, **39:** 2853(1996).

[6] S. Sharidan and I. Pop, Heat and Mass Transfer, **32:** 657(2005).

[7] S. Biringen, and G Danabasogluf, *J. Thermo Physics*, **4**(3)**:** 357(1990).

[8] T.C. Jue and B. Ramaswamy, Heat and Mass Transfer, **38:** 665(2001).

[9] Y. Shu, B. Q. Li, and H.C. De Groh, Numerical Heat Transfer, **39:** 245 (2001).

[10] P.G. Siddheshwar and S. Pranesh, *Journal of Magnetism and Magnetic material*, **192**(1)**:** 159(1999).